Math 53, Discussions 116 and 118 **2022-10-21**

Double and triple integrals

Questions

Question 1. A lamina of constant density ρ occupies the half-disk $x^2 + y^2 \leq 1$, $x \geq 0$. Find its center of mass. Feel free to use geometry where applicable.

Question 2. Let

$$f(x) = \int_x^1 \cos(t^2)\,\mathrm{d}t.$$

Note that this is a function of x, *not* of t. Find the average value of f on the interval $[0, 1]$.

(The average value of a function over some region can be computed by integrating the function over that region, and then dividing by the size of the region.)

Question 3. Rewrite the following integrals in the five other integration orders.

(a) $\int_0^1 \int_y^1 \int_0^y f(x, y, z)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$
(b) $\int_0^1 \int_y^1 \int_0^z f(x, y, z)\,\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}y$

Question 4. Show that

$$\int_{-1}^1 \int_0^2 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \arctan(x\sqrt{y}\cos(z))e^{-zx^2}\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x = 0$$

by changing the order of integration so that x is done first.